Sample Paper 4

Leaving Certificate

Mathematics

Paper 2

Higher Level

Time: 2 hours, 30 minutes

300 marks

Examination number

Centre stamp

Running total	

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Grade

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions.

Write your answers in the spaces provided in this booklet. You will lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here: _____

Section A — Concepts and Skills — 150 marks

Answer **all six** questions from this section.

Question 1 **(25 marks)**

(a) Find the acute angle between the lines $x + y - 3 = 0$ and $2x + y - 1 = 0$, to one decimal place.

(b) Find the equations of the lines making an angle of $\tan^{-1} 3$ with the line $2x - y - 8 = 0$ and passing through $(4, -1)$.

Question 2 **(25 marks)**

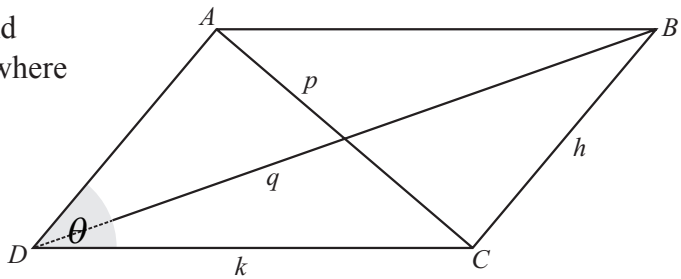
Find the equation of a circle through the centre of the circle, $s: x^2 + y^2 - 2x - 4y - 20 = 0$, which touches it internally at (–2, 6).

Question 3 **(25 marks)**

(a) The adjacent interior angles in a parallelogram add up to _____ .

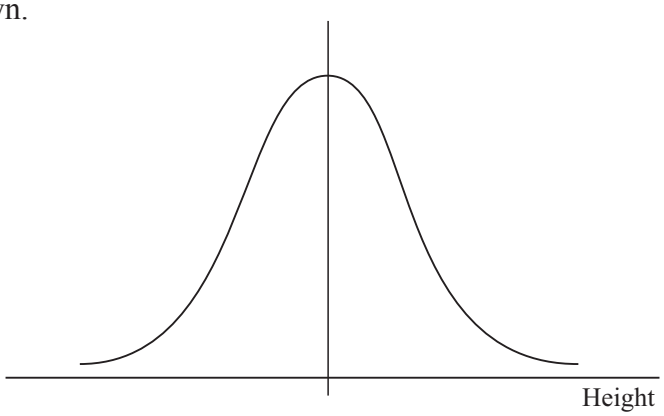
(b) Show that $\cos(180^\circ - \theta) = -\cos\theta$.

(c) Use the Cosine rule for triangles ACD and BDC to show that $p^2 + q^2 = 2h^2 + 2k^2$, where $|AC| = p$, $|DB| = q$, $|DC| = k$ and $|BC| = h$.



Question 4 **(25 marks)**

The average height μ of a group of students is 175 cm, with an upper quartile (UQ) value of 180 cm. The heights follow a normal distribution as shown.



(a) Find the values of:

(i) the median M,
$M =$ _____

(ii) the lower quartile (LQ),
$LQ =$ _____

(iii) the interquartile range.
Interquartile range = _____

On the curve shown, mark in the median M, the lower quartile (LQ) and the upper quartile (UQ).

(b) What percentage of the students have a height less than 180 cm? Shade this area on the curve above.

PERCENTAGE OF STUDENTS = _____

(c) Find the standard deviation of this distribution.

(d) Show, by finding z values, that the probability a student's height lies within the interquartile range is 0.5.

Question 5 **(25 marks)**

(a) The probability distribution of the number of students in a Geography class who forget their textbook on a given day is shown below:

x	1	2	3	4	5
$P(x)$	0·1	0·2	0·3	0·3	0·1

(i) Find the probability that at least three students forget their textbook on a given day.

(ii) Find the mean number of students who forget their textbook on a given day.

(b) A group of 300 people were surveyed as to whether they were for or against an EU treaty. The table below summarises their responses. Complete the table.

Gender	For	Against	Total
Male	58		143
Female		73	
Total			

(i) Find the probability that a person is male *and* against the treaty.

(ii) Find the probability that a person is female *or* is for the treaty.

(iii) Given that a person is for the treaty, what is the probability that this person is male?

Question 6 **(25 marks)**

(a) A triangle ABC has vertices: $A(0, 0)$, $B(1, 4)$ and $C(4, 1)$. Find the area of the triangle.

(b) An enlargement of triangle ABC is constructed with scale factor $k = \frac{3}{2}$ and centre $(-1, 3)$ giving a new triangle $A'B'C'$. Find the co-ordinates of $A'B'C'$.

(c) Find the area of triangle $A'B'C'$ and show that $\dfrac{\text{Area triangle } A'B'C'}{\text{Area triangle } ABC} = k^2$.

Section B Contexts and Applications 150 marks

Answer Question 7, Question 8, and Question 9.

Question 7 **(50 marks)**

(a) **(i)** What is meant by a random variable?

(ii) There are two types of random variable. What are they? Give an example of each.

(iii) Define the expected value of a probability distribution.

(b) **(i)** What is a prime number?

(ii) Two dice, *A* and *B*, are rolled. Complete the following sample space diagram for the sum of the values showing on the faces.

Die *A*

Die *B*	1	2	3	4	5	6
1	2					
2						
3						
4						
5						
6						

(iii) What is the probability of a sum that is a prime number?

(iv) What is the probability that the sum is not a prime number?

(c) Bob and Peter play a game based on rolling two dice.
The game is that Bob pays Peter €3 if the sum on the faces is not a prime number and Peter pays Bob €3 if the sum is a prime number.

Let x be a random variable that represents the net income to Bob. Complete the table:

Outcome	Not a Prime Sum	Prime Sum
P		
Net income to Bob		
$xP(x)$		

(d) **(i)** What is the expected value of the game to Bob? What does this value mean?

(ii) If the game was played 30 times, what are Bob's expected winnings/losses?

(e) If the sum is prime, what should Peter pay Bob in order that Bob's expected value is 50 cent?

Question 8 **(25 marks)**

A man at a casino craps table believes one of the die to be biased in favour of the number 4.
The casino agrees to a test at a 5% significance level rolling the die 20 times. The results are:

4	6	2	3	5
4	2	3	2	1
4	4	4	1	6
5	1	5	6	4

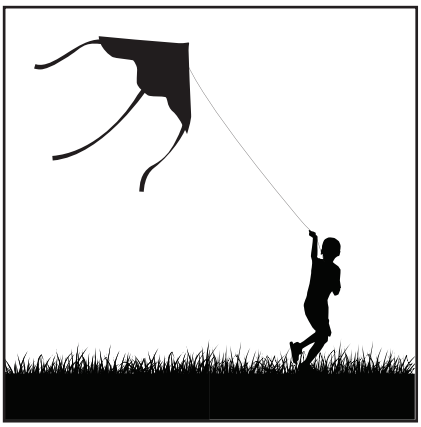
The number 4 appears six times in this test, which seems unusual in that it occurs more often than the other numbers.

(a) Find the probability that the 4 appears at least six times in 20 rolls of a fair die.

(b) If the probability is greater than or equal to 5%, make a conclusion about the die.

Question 9 **(75 marks)**

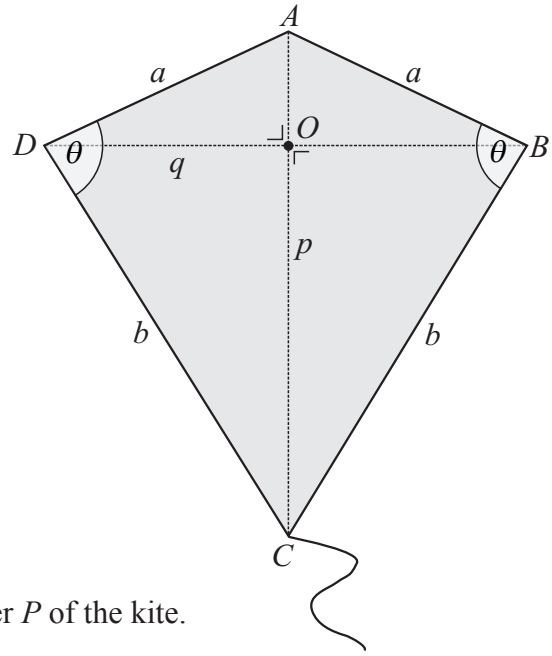
A kite is made up of two crossed sticks (diagonals) over which the material is stretched. A kite is a quadrilateral with special properties, as shown in the diagram (below right).



1. SIDES: It has two pairs of equal sides. Adjacent sides are equal in length.
$|DA| = |AB| = a$, $|DC| = |CB| = b$

2. ANGLES: The angles are equal where the two pairs of equal sides meet.
$|\angle ADC| = |\angle ABC| = \theta$

3. DIAGONALS: Diagonals (dashed lines) cross at right angles, and one of the diagonals bisects (cuts equally in half) the other.
Diagonal 1: $|AC| = p$
Diagonal 2: $|DB| = q$
$AC \perp DB$ and $|DO| = |OB| = \frac{1}{2}q$



(a) Name the type of shape formed if:

(i) the four sides of the kite are equal in length,

SHAPE: _____

(ii) the four angles in the kite are equal in measure.

SHAPE: _____

(b) **(i)** Write down a formula for calculating the perimeter P of the kite.

Perimeter P = _____

(ii) Write a formula for calculating the area of the kite using **1**. the diagonals, and **2**. the sides and θ.

1. Diagonals	**2**. Sides

(c) A kite is made from the following measurements: $|AB| = |AD| = 30$ cm, $|BC| = |CD| = 40$ cm, $|\angle ADC| = 100^{\circ}$. Draw a sketch of the kite including the measurements.

(i) Find $|AC|$, the length of one diagonal of the frame.

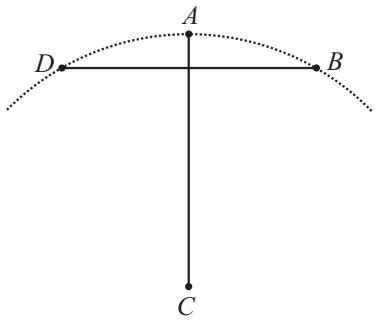
KITE DIAGRAM

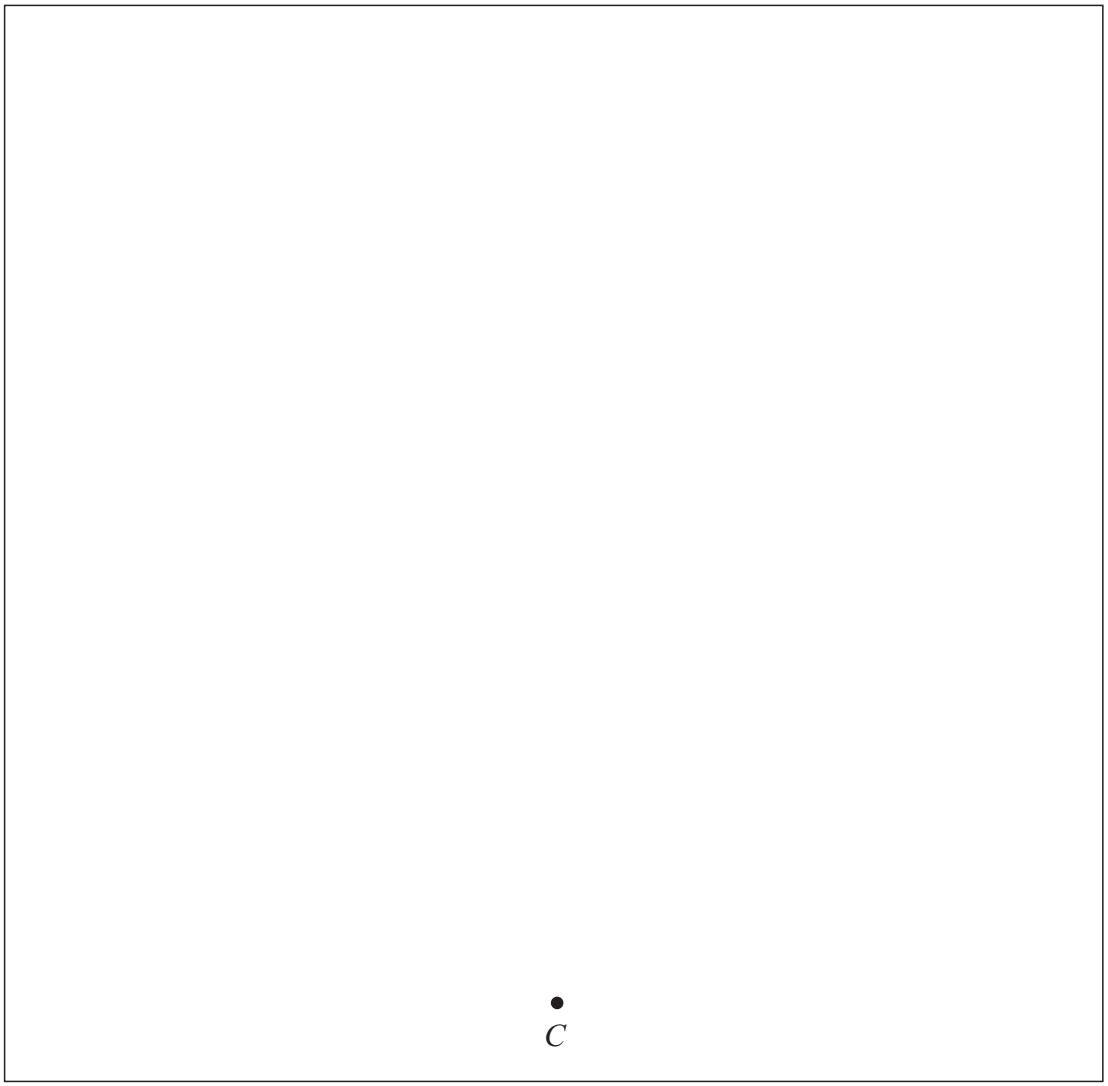
(ii) Find $|\angle CAD|$, to one decimal place.

(iii) Find $|BD|$, to one decimal place.

(iv) Find the area of the kite, to the nearest centimetre squared.

(d) Paul is constructing a kite by following these instructions: his kite is equidiagonal which means that both diagonals, $[AC]$ and $[DB]$, are equal in length.
Using point C as centre, he draws an arc of radius of length $|AC|$, as shown to the right. He draws the other diagonal at right angles to the first diagonal so that each end touches the arc. He then joins up the points $ABCD$.
Using a compass and ruler, draw an equidiagonal kite below, with diagonals of length 10 cm, using C as the centre of your arc.

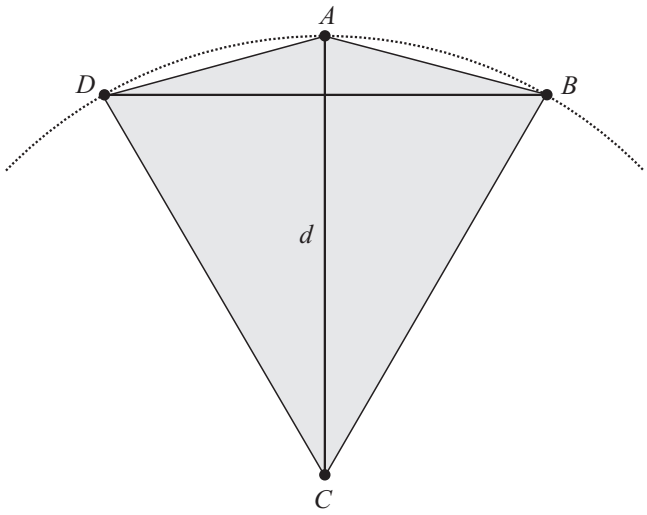




(e) **(i)** Show that the measure of the angle $|\angle ABD|$ in the equidiagonal kite below is given by:

$$|\angle ABD| = \tan^{-1}(2-\sqrt{3}) = 15^{\circ}.$$

In your analysis, denote the length of the diagonal by d, such that $|AC| = |DB| = d$. DAB is an arc from a circle whose centre is C.



(ii) Hence, write down the measure in degrees of the four interior angles of the kite.

$|\angle ADC| =$ _____ $|\angle ABC| =$ _____

$|\angle BAD| =$ _____ $|\angle BCD| =$ _____

(f) Using the values of the angles on page 13 of the *Formulae and Tables* booklet, prove that $\tan 15^{\circ} = (2-\sqrt{3})$. Fully derive the result without using a calculator. Show all your steps clearly.